Comment I on "Resonant and antiresonant frequency dependence of the effective parameters of metamaterials"

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Noting that the imaginary parts of the permittivity and permeability of any linear, homogeneous, passive, dielectric-magnetic material are always positive, independent of the signs of their real parts, we conclude that recent claims by Koschny *et al.* [Phys. Rev. E **68**, 065602 (2003)] are unphysical.

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The purpose of this Comment is to point out that, contrary to the unphysical conclusion obtained by Koschny *et al.* [1], the imaginary parts of the permittivity and permeability of any linear, homogeneous, passive, dielectric-magnetic material are always positive, independent of the signs of their real parts. An underlying assumption is that the time dependence is of the form $\exp(-i\omega t)$, where $i=\sqrt{-1}$, ω is the angular frequency, and t denotes the time. We must also bear in mind that the frequency-domain constitutive parameters of a nonaging, linear, homogeneous material continuum must be independent (i) of the electromagnetic field phasors excited therein, as well as (ii) of the geometric configuration of the system employing that continuum.

The argument advanced in Ref. [1] centers on Eq. (10) of that paper,

$$W = \frac{1}{4\pi}\int d\omega\ \omega[\varepsilon''(\omega)|\vec{E}(\omega)|^2 + \mu''(\omega)|\vec{H}(\omega)|^2]. \qquad (1)$$

Here, W is the dissipated energy, $\vec{E}(\omega)$ and $\vec{H}(\omega)$ are the electric and the magnetic field phasors, while $\varepsilon''(\omega)$ and $\mu''(\omega)$ are the imaginary parts of the relative permittivity and relative permeability. In Ref. [1], it is stated that "$W > 0$ does not require that ε'' and μ'' must be simultaneously positive." This statement is untrue.

It is better to rewrite the foregoing equation to indicate spatial dependence; thus,

$$W(\vec{r}) = \frac{1}{4\pi}\int d\omega\ \omega[\varepsilon''(\omega)\Phi(\vec{r},\omega) + \mu''(\omega)]|\vec{H}(\vec{r},\omega)|^2. \qquad (2)$$

While working only with one plane wave, it is easy to conclude that the ratio

$$\Phi(\vec{r},\omega) = \frac{|\vec{E}(\vec{r},\omega)|^2}{|\vec{H}(\vec{r},\omega)|^2} \qquad (3)$$

is fixed for all $\vec{r}$. But, in general, it is not. For a specific ω, $\Phi(\vec{r},\omega) \in [0,\infty)$ for an electromagnetic field that is not a single plane wave.

A commonplace example is furnished by the TE_{mn} mode in a rectangular waveguide. Expressions for the modal fields are available in textbooks; see, e.g., Ref. [2]. The presence of the longitudinal component of the magnetic field phasor in the TE_{mn} mode means that $\Phi(\vec{r},\omega)$ varies over the transverse cross section of the waveguide, whether it is hollow or filled with a homogeneous material.

Now, it is true that $W(\vec{r}) > 0$ in real materials. This condition implies that the integrand on the right side of Eq. (2) must be positive, regardless of the value of $\Phi(\vec{r},\omega) \in [0,\infty)$. That can happen only if $\varepsilon''(\omega) > 0$ and $\mu''(\omega) > 0$, which contradicts the statement proffered in Ref. [1].

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[1] T. Koschny, P. Markoš, D. R. Smith, and C. M. Soukoulis, Phys. Rev. E **68**, 065602 (2003).

[2] J. D. Kraus, *Electromagnetics* (McGraw-Hill, New York, 1984), pp. 544–550.

